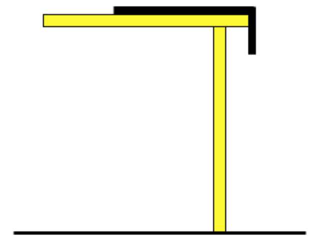
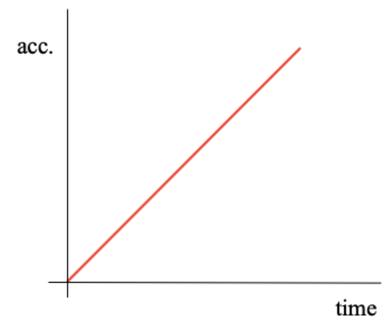
Inspired by "Frensley Physics" Facebook page;



Rope problem: A rope of length L and mass m is placed on a frictionless table so that a quarter of it hangs over the edge. At t = 0, the rope is released and begins to slide off the table. You are asked to graph the acceleration of the hanging end of rope, assuming the rope-end's initial position is at L/4. Your graph is shown below.



a.) A student, someone who wants to curry favor with you, looks at the graph and notices one thing that is right about it. Explain with justification what that one thing is.

--the acceleration will be due to the gravitational force on the rope mass m that is hanging over the edge of the table;
--as the amount of rope hanging over the table will change with time, mg will increase with time;
--because the net force will be increasing, the acceleration will be increasing;

b.) While we are at it, what information were you given that is completely useless, given your task? Explain.

--being told that the end of the hanging rope is initially at L/4 makes no sense as there is no axis on the sketch.

c.) A second less obsequious student looks at the graph and finds two things wrong with it. Identify (with justification) what those two are.

--first goof: the acceleration is not zero at t = 0;
--second goof: the force is proportional to the amount of rope over the edge;
  --the amount of rope over the edge is NOT increasing linearly as the rope is accelerating;
  --the acceleration is proportional to the force, which means the acceleration should not be increasing linearly;
  --in fact, the curve should be curved upward as more and more rope makes it over the edge with time.
--a picky point: assuming we are using the standard "up is positive" criterion, the acceleration of the rope should have been negative, not positive;

d.) The situation changes some. Now the rope starts with just a tiny bit of its length hanging over the tabletop. It is released from rest and allowed to fall. Once the last bit of it has left the table, it's velocity is observed to be "v." Two students attempt to analyze the situation. Their work is shown below. Both are wrong. For both, explain where they went astray.

Approach 1:

--principle: gravitational potential energy transformed into kinetic energy:

--bottom-end changes height by $\Delta y = L$;

$$\frac{1}{2}Mv^2 = MgL$$

$$\Rightarrow v = (2gL)^{1/2}$$

Approach 2:

--Rope's weight = Mg;

--Force acts as rope travels distance L:

--Work = Fd, so work = MgL;

--Work = gain in KE, so $\frac{1}{2}Mv^2 = MgL$

$$\Rightarrow v = (2gL)^{1/2}$$

Approach 1:

--for potential energy to be used properly, the change of the rope's position must track the rope's *center of mass*, so delta y is not L, it's L/2;

Approach 2:

--the work calculation is treating the force on the rope as a constant;

--in fact, the force on the rope depends upon how much rope is hanging over the edge at a particular point in time;

--so the work calculation should be executed by integrating Fdy over the length of the rope.